The Golden Digits International Contest
9th Edition, February 2025

Problem 1. Find all positive integers n such that

$$1^n + 2^n + \ldots + n^n$$

is a prime number.

Problem 2. Let $\triangle ABC$ be an acute triangle. Let I be its incenter, $D = AI \cap (ABC)$ and E, $F \in (BIC)$ such that A, E, F are collinear. Let $E_b \in AB$ such that $EE_b = E_bB$. E_c is defined similarly. Let $K \in E_bE_c$ such that $DK \parallel EF$. Prove that $AK \cap BC \cap DF \neq \emptyset$.

Problem 3. A and B play a game on an $n \times n$ board. On each turn, A places a rook on an empty square of the board, and B moves it to a neighboring square (two squares are neighbors if they share a common edge). If all neighboring squares are occupied, the rook remains in place. A wins when there are n rooks on the board that do not attack each other. Determine for which constants $c \in \mathbb{R}$ there exists $c_0 \in \mathbb{R}$ such that A can win in at most $cn + c_0$ moves for all $n \in \mathbb{N}$.

Time allowed: 4 hours and 30 minutes
Each problem is worth 7 points